Higher-Order Corrections to the Acoustic Attenuation in n-Ge[†]

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Higher-order corrections of the order of $\hbar\omega_{q\lambda}/4\Delta$ and $\hbar\omega_{q\lambda}/K_B T$ used in Kwok's results are observed to explain the acoustic-attenuation Sb-, P-, and As-doped Ge samples at temperatures below 10 °K.

Kwok's results[1,2] have been used to explain the measurements of the acoustic attenuation in n-Ge by Pomerantz.[3] The agreement between theory and experiment is excellent for As- and P-doped Ge samples. The calculations[3b] for Sb-doped Ge, however, show that there is a slight deviation at temperatures below 10 °K. Kwok (Ref. 1, Fig. 2), in his calculations for P-doped Ge, has also observed deviation at temperatures below 10 °K. To explain these deviations below 10 °K for Sb- and P-doped Ge samples, it is desirable to consider the higher-order corrections of the order $\hbar\omega_{q\lambda}/4\Delta$ and $\hbar\omega_{q\lambda}/K_B T$. In the present work,[4] we modify Kwok's results by considering higher-order corrections to explain the disagreement between theory and experiment below 10 °K. The values of $\hbar\omega_{q\lambda}/4\Delta$ and $\hbar\omega_{q\lambda}/K_B T$ clearly show that one should consider the higher-order corrections in calculating the acoustic attenuation of n-Ge.

Equation (15) of Ref. 1 can be written as

$$\alpha_{q\lambda}=\frac{\pi}{2\rho C^3}\,\omega_{q\lambda}\sum_{q'\lambda'}\frac{\omega_{q'\lambda'}}{C^5_{\lambda'}}\,\delta(\omega_{q\lambda}-\omega_{q'\lambda'})\Bigg[f_0(T)\times\left|\sum_m\left(\frac{\Xi^{\lambda'}_{om}(\vec{q}\,')\,\Xi^{\lambda}_{mo}(\vec{q})}{4\Delta-\hbar\omega_{q\lambda}}+\frac{\Xi^{\lambda}_{om}(\vec{q})\,\Xi^{\lambda'}_{mo}(\vec{q}\,')}{4\Delta+\hbar\omega_{q\lambda}}\right)\right|^2 +f(T)\sum_{n,n'}\left|\left(\frac{\Xi^{\lambda'}_{n'o}(\vec{q}\,')\,\Xi^{\lambda}_{on}(\vec{q})}{4\Delta+\hbar\omega_{q\lambda}}+\frac{\Xi^{\lambda}_{n'o}(\vec{q})\,\Xi^{\lambda'}_{on}(\vec{q}\,')}{4\Delta+\hbar\omega_{q\lambda}}\right)\right|^2\Bigg], \tag{1}$$

where each term occurring in the equation is defined in the usual manner.[1,5] For $\hbar\omega_{q\lambda}<4\Delta$, we can expand $(1\pm\hbar\omega_{q\lambda}/4\Delta)^{\pm 1}$ by the binomial theorem. For elastic scattering of phonons, we[4] can further simplify Eq. (1) to give

$$\alpha_{q\lambda}=10\,\frac{(E_u/3)^4}{4\pi\rho^2C^8(4\Delta)^2}\,F^4(q)\,\omega^4_{q\lambda}\,\langle\langle D^{\lambda 1'}_s\rangle\rangle\times\left[[f_0(T)+2f(T)]+\left(\frac{\hbar\omega_{q\lambda}}{4\Delta}\right)^2[2f_0(T)+5f(T)]+\left(\frac{\hbar\omega_{q\lambda}}{4\Delta}\right)^4[3f_0(T)+8f(T)]+\cdots\right]. \tag{2}$$

Equation (2) indicates that the elastic scattering of phonons is proportional to the phonon frequency to its fourth power. Kwok has shown that elastic scattering of phonons contributes negligibly to acoustic attenuation. Equation (1) is now modified for inelastic scattering of phonons (considering the higher-order corrections of the order $\hbar\omega_{q\lambda}/4\Delta$ and $\hbar\omega_{q\lambda}/K_B T$) as

$$\alpha_{q\lambda}=\frac{(E_u/3)^4}{4\pi\rho^2C^3}\,\frac{1}{\hbar K_B T}\left(1+\frac{\hbar\omega_{q\lambda}}{4\Delta}\right)^3\left(\frac{4\Delta}{\hbar}\right)^3\times f\left(\frac{\hbar\omega_{q\lambda}}{K_B T}\right)F^2(q)\sum_{\lambda'}\frac{F^2(q')}{C^5_{\lambda'}}\,E^{\lambda\lambda'}, \tag{3}$$

where

$$f\left(\frac{\hbar\omega_{q\lambda}}{K_BT}\right)=\left[1-\frac{1}{2}\,\frac{\hbar\omega_{q\lambda}}{K_BT}+\frac{1}{6}\left(\frac{\hbar\omega_{q\lambda}}{K_BT}\right)^2-\frac{1}{24}\left(\frac{\hbar\omega_{q\lambda}}{K_BT}\right)^2+\cdots\right],$$

$$E^{\lambda\lambda'}=\left[M^{\lambda\lambda'}-2\left(\frac{\hbar\omega_{q\lambda}}{4\Delta}\right)N^{\lambda\lambda'}+\left(\frac{\hbar\omega_{q\lambda}}{4\Delta}\right)^2\times(M^{\lambda\lambda'}+2N^{\lambda\lambda'})-2\left(\frac{\hbar\omega_{q\lambda}}{4\Delta}\right)^3(M^{\lambda\lambda'}+N^{\lambda\lambda'})+\left(\frac{\hbar\omega_{q\lambda}}{4\Delta}\right)^4(3M^{\lambda\lambda'}+2N^{\lambda\lambda'})-\cdots\right],$$

$$M^{\lambda\lambda'}=\textstyle\sum_n\left(\sum_m D^{\lambda}_{om}\,D^{\lambda'}_{mn}\right)^2,$$

$$N^{\lambda\lambda'}=\textstyle\sum_n\left(\sum_m D^{\lambda}_{om}\,D^{\lambda'}_{mn}\,D^{\lambda'}_{om}\,D^{\lambda}_{mn}\right).$$

Kwok[1] has shown that inelastic scattering of phonons mainly contributes to the acoustic attenuation. Comparing Eq. (22) of Ref. 1 and Eq. (3), we obtain

$$\frac{(\alpha_{q\lambda})_{\text{present}}}{(\alpha_{q\lambda})_{\text{Kwok}}}=\left(1+\frac{\hbar\omega_{q\lambda}}{4\Delta}\right)^3 f\left(\frac{\hbar\omega_{q\lambda}}{K_BT}\right)\sum_{\lambda'}\frac{E^{\lambda\lambda'}}{M^{\lambda\lambda'}}. \tag{4}$$

The values of $(1+\hbar\omega_{q\lambda}/4\Delta)^3$ and $\sum_{\lambda'}(E^{\lambda\lambda'}/M^{\lambda\lambda'})$ are temperature independent and depend upon the value of 4Δ. The temperature-dependent term is $f(\hbar\omega_{q\lambda}/K_B T)$. The variation of $(\alpha_{q\lambda})_{\text{present}}/(\alpha_{q\lambda})_{\text{Kwok}}$ with temperature is shown in Fig. 1, for Sb-, P-, and As-doped samples. Figure 1 clearly shows that the value of $(\alpha_{q\lambda})_{\text{present}}/(\alpha_{q\lambda})_{\text{Kwok}}$ sharply reduces up to 5 °K, its sharp reduction is slowed down as temperature further increases,

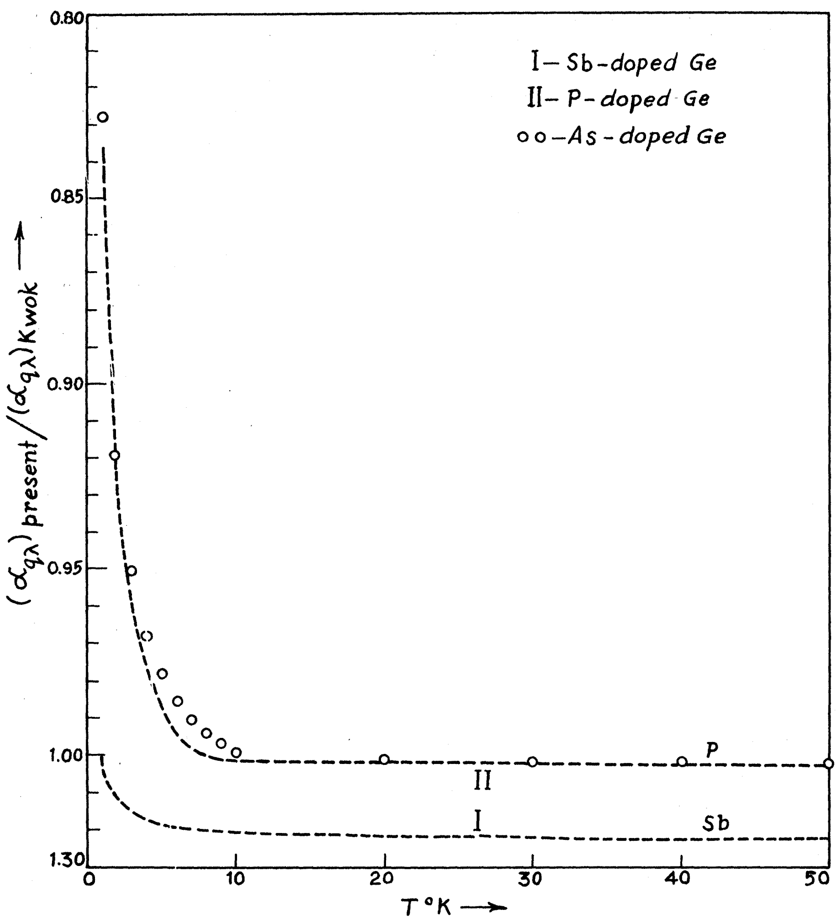

FIG. 1. Plot of $(\alpha_{q\lambda})_{\text{present}}/(\alpha_{q\lambda})_{\text{Kwok}}$ vs temperature for Sb-, P-, and As-doped Ge samples. Dotted curves correspond to Sb- and P-doped Ge samples and circles to As-doped Ge sample.

and after 10 °K there remains no variation in the values of the curve of $(\alpha_{q\lambda})_{\text{present}}/(\alpha_{q\lambda})_{\text{Kwok}}$ vs temperature. From Fig. 1 we find that calculated values of $(\alpha_{q\lambda})_{\text{present}}$ compared to $(\alpha_{q\lambda})_{\text{Kwok}}$ are reduced for temperatures $T < 10$ °K when we consider higher-order corrections in Kwok's results. After 10 °K, the $(\alpha_{q\lambda})_{\text{present}}$ equalizes $(\alpha_{q\lambda})_{\text{Kwok}}$. This gives better agreement for the temperatures where donor-electron–phonon interactions dominate the other phonon scattering process.[4]

The present calculated results for acoustic attenuation, therefore, suggest that to explain the deviation between theory and experiment at low temperatures one should consider the higher-order corrections of the order of $\hbar\omega_{q\lambda}/4\Delta$ and $\hbar\omega_{q\lambda}/K_B T$.

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[4]Details of the present paper will be published elsewhere.

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